

On the Performance of IRS-Assisted Relay Systems

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Introduction

- An **intelligence reflecting surface (IRS)** having a large number of tiny passive reflectors can enable a controllable wireless propagation environment by introducing distinct delays to the reflected electromagnetic (EM) waves.
- These delays in turn result in controllable phase-shifts, which can be used to intelligently reconfigure propagation properties of EM waves through the wireless medium.
- This feature of IRSs can be utilized to improve the signal-to-noise ratio (SNR) of an end-to-end communication between a transmitter and a receiver by enabling constructive additions of EM waves at a desired destination.
- **Relay-assisted** cooperative communications have been studied for well over two decades due to their potential of enhancing the performance of wireless systems
- Relaying can effectively reduce the end-to-end path-loss in terms of shorter-hop distances and amplify-and-forward (AF) or decode-and-forward (DF) operations at intermediate relays.

Motivation and contribution

- The fundamental performance metrics of the IRS-relay cascaded communication systems have not yet been investigated in the open literature.
- Our objective is to develop an analytical framework of an IRS-assisted relay system by deriving the performance bounds pertaining to the proposed IRS-relay cascaded system.
- First, the end-to-end optimal SNR is probabilistically characterized by tightly approximating it by a mathematically tractable counterpart by invoking the central limit theorem (CLT).
- Thereby, a tight upper bound for the cumulative distribution function (CDF) of this approximated optimal SNR is derived.
- By using this CDF, tight bounds/approximations for the average achievable rate, SNR/rate outage probability, and average SER are derived in closed-form.
- Then, the tightness of our performance bounds/approximations is validated through Monte-Carlo simulations.
- Finally, a set of insightful numerical results is presented to explore the performance gains of the proposed IRS-assisted relay system.

System and channel model

System model:

- A single-antenna source (S)
- A single-antenna destination (D)
- A single-antenna relay (R)
- An IRS having N reflectors

Channel model:

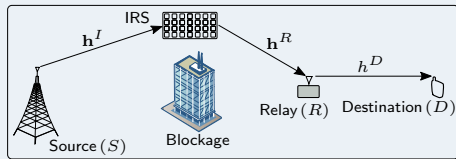
- h_n^I : the channel between S and the n th reflector of the IRS
- h_n^R : the channel between the n th reflector and R
- h^D : the channel between R and D

The polar-form of these channels

$$u = \beta_u e^{j\theta_u},$$

where $u \in \{h_n^I, h_n^R, h^D\}$ and $n \in \mathcal{N}$.

IRS-aided cell-free set-up



- β_u denotes the envelop of u , and θ_u is the phase of u .
- β_u is assumed to be independent Rayleigh distributed as

$$f_{\beta_u}(x) = (x/\xi_u) \exp(-x^2/(2\xi_u)),$$

where $\xi_u = \zeta_u/2$ is the Rayleigh parameter, and ζ_u accounts for the large-scale fading/path-loss of the channel u .

Signal model

- The signal transmitted by S reaches D through the IRS- R cascaded channel.
- The signal received at R during the first time-slot as

$$y_R = \sqrt{P}(\mathbf{h}^R)^T \mathbf{\Theta} \mathbf{h}^I x + w_R,$$

- x : the transmit signal from S satisfying $\mathbb{E}[|x|^2] = 1$
- P : the transmit power at S
- $w_R \sim \mathcal{CN}(0, \sigma_{w_R}^2)$: AWGN at R
- $\mathbf{h}^I = [h_1^I, \dots, h_n^I, \dots, h_N^I]^T \in \mathbb{C}^{N \times 1}$
- $(\mathbf{h}^R)^T = [h_1^R, \dots, h_n^R, \dots, h_N^R] \in \mathbb{C}^{1 \times N}$
- $\mathbf{\Theta} = \text{diag}(\eta_1 e^{j\theta_1}, \dots, \eta_n e^{j\theta_n}, \dots, \eta_N e^{j\theta_N}) \in \mathbb{C}^{N \times N}$: the reflective properties of the IRS, $\eta_n e^{j\theta_n}$, represents the complex-valued reflection coefficient of the n th reflector of the IRS.
- By exploiting the properties of $\mathbf{\Theta}$, the rearranged received signal at R

$$y_R = \sqrt{P} \sum_{n \in \mathcal{N}} h_n^R \eta_n e^{j\theta_n} h_n^I x + w_R.$$

Signal model continued

- During the second time-slot, R first amplifies its received signal and then forwards it towards D .
- the signal received at D can be written as

$$y_D = Gh^D y_R + w_D = \sqrt{P} Gh^D \sum_{n \in \mathcal{N}} h_n^R \eta_n e^{j\theta_n} h_n^I x + Gh^D w_R + w_D,$$

where $w_D \sim \mathcal{CN}(0, \sigma_{w_D}^2)$ is an AWGN at D .

- G denotes the relay amplification factor, which is designed to constraint the instantaneous transmit power (P_R) at R

$$G = \sqrt{P_R / \left(P \left| \sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I} e^{j\phi_n} \right|^2 + \sigma_{w_R}^2 \right)},$$

where $\phi_n = \theta_n + \theta_{h_n^R} + \theta_{h_n^I}$.

- the received SNR at D as

$$\gamma = \frac{P |Gh^D \sum_{n \in \mathcal{N}} h_n^R \eta_n e^{j\theta_n} h_n^I|^2}{|Gh^D|^2 \sigma_{w_R}^2 + \sigma_{w_D}^2}.$$

Signal model continued

- This SNR in terms of the channel phases

$$\gamma = \frac{P \left| G \beta_{h^D} e^{j\theta_{h^D}} \sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I} e^{j\phi_n} \right|^2}{\left| G \beta_{h^D} e^{j\theta_{h^D}} \right|^2 \sigma_{w_R}^2 + \sigma_{w_D}^2}.$$

- We maximize the received SNR at D by smartly adjusting the phase-shifts (θ_n) at each reflector to enable constructive addition of the signal terms inside the summation.
- The optimal choice of θ_n to maximize the received SNR

$$\theta_n^* = \underset{-\pi \leq \theta_n \leq \pi}{\operatorname{argmax}} \gamma = -(\theta_{h_n^R} + \theta_{h_n^I}), \quad \text{for } n \in \mathcal{N}.$$

- The optimal SNR at D

$$\gamma^* = \frac{P (G^*)^2 \beta_{h^D}^2 \left(\sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I} \right)^2}{(G^*)^2 \beta_{h^D}^2 \sigma_{w_R}^2 + \sigma_{w_D}^2} = \frac{\bar{\gamma}_R \bar{\gamma}_D \beta_{h^D}^2 \left(\sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I} \right)^2}{\bar{\gamma}_R \left(\sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I} \right)^2 + \bar{\gamma}_D \beta_{h^D}^2 + 1},$$

where $\bar{\gamma}_R = P/\sigma_{w_R}^2$ and $\bar{\gamma}_D = P/\sigma_{w_D}^2$.

Preliminary analysis

- The optimal received SNR is probabilistically characterized by deriving a tight approximate to its CDF.
- First, we define $Z = \sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I}$.
- By using the fact that the envelopes $\beta_{h_n^R}$ and $\beta_{h_n^I}$ are independent Rayleigh distributed random variables, Z is closely approximated by an one-sided Gaussian distributed random variable (\tilde{Z}) by invoking the CLT

$$f_Z(y) \approx f_{\tilde{Z}}(y) = \frac{\psi}{\sqrt{2\pi\sigma_Z^2}} \exp\left(\frac{-(y - \mu_Z)^2}{2\sigma_Z^2}\right), \text{ for } y \geq 0.$$

- $\psi \triangleq 1/\mathcal{Q}(-\mu_Z/\sigma_Z)$: normalization factor, $\int_{-\infty}^{\infty} f_{\tilde{Y}}(x)dx = 1$
 - $\mu_Z = \sum_{n \in \mathcal{N}} \pi \eta_n \left(\xi_{h_n^R} \xi_{h_n^I}\right)^{1/2} / 2$
 - $\sigma_Z^2 = \sum_{n \in \mathcal{N}} \eta_n^2 \xi_{h_n^R} \xi_{h_n^I} (16 - \pi^2) / 4$
- We define γ_R to be

$$\gamma_R = \bar{\gamma}_R Z^2 = \bar{\gamma}_R \left(\sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I} \right)^2.$$

Preliminary analysis continued

- A tight approximation for the PDF of γ_R

$$f_{\gamma_R}(x) \approx \frac{\psi}{2\sqrt{\pi}\sigma_R^2 x} \exp\left(\frac{-(\sqrt{x} - \mu_R)^2}{2\sigma_R^2}\right), \text{ for } x \geq 0,$$

where $\mu_R = \sqrt{\bar{\gamma}_R}\mu_Z$ and $\sigma_R^2 = \bar{\gamma}_R\sigma_Z^2$.

- An approximated CDF for γ_R

$$F_{\gamma_R}(x) \approx 1 - \psi \mathcal{Q}\left((\sqrt{x} - \mu_R)/\sigma_R\right), \text{ for } x \geq 0.$$

- Next, we define γ_D to be $\gamma_D = \bar{\gamma}_D\beta_{h^D}^2$. The CDF of γ_D

$$F_{\gamma_D}(x) = 1 - \exp(-x/\sigma_D^2), \text{ for } x \geq 0,$$

where $\sigma_D^2 = \bar{\gamma}_D\zeta_{h^D}$.

- Since the exact derivation of the CDF of γ^* appears to be mathematically involved, we resort to an asymptotically exact upper bound

$$\gamma^* \approx \tilde{\gamma}^* = \min\left(\bar{\gamma}_R\left(\sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I}\right)^2, \bar{\gamma}_D\beta_{h^D}^2\right).$$

Preliminary analysis continued

- By noticing that $\tilde{\gamma}^* = \min(\gamma_R, \gamma_D)$, the approximated CDF of γ^* (or the exact CDF of $\tilde{\gamma}^*$)

$$\begin{aligned} F_{\gamma^*}(y) &\approx F_{\tilde{\gamma}^*}(y) = 1 - (1 - F_{\gamma_R}(y))(1 - F_{\gamma_D}(y)) \\ &= 1 - \psi \mathcal{Q}((\sqrt{y} - \mu_R)/\sigma_R) \exp(-y/\sigma_D^2). \end{aligned}$$

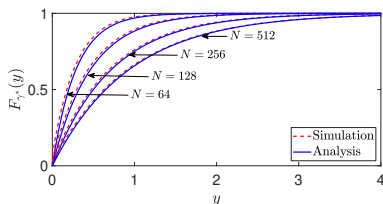


Figure: The CDF of SNR (γ^*) for $N \in \{64, 128, 256, 512\}$ and $\bar{\gamma} = 10$ dB.

Figure illustrates that our analytical CDF approximation is accurate for medium-to-large numbers of reflective elements (N) at the IRS. A relatively larger N is practically feasible and cost effective for IRSs, and hence, our probabilistic characterization of the optimal SNR is useful in deriving performance bounds for the cascaded IRS-relay channels.

Average Achievable Rate

- The average achievable rate of the proposed system

$$\mathcal{R} = \mathbb{E} \left[\frac{1}{2} \log_2 (1 + \gamma^*) \right],$$

where the pre-log factor of $1/2$ is due to the fact that half-duplex relay mode requires two time-slots for end-to-end data transmission for the proposed system model.

- The exact derivation of \mathcal{R} seems mathematically intractable, and hence, a tight upper bound by invoking the Jensen's inequality

$$\mathcal{R} \leq \mathcal{R}_{ub} = \frac{1}{2} \log_2 (1 + \mathbb{E}[\gamma^*]) \approx \frac{1}{2} \log_2 (1 + \mathbb{E}[\tilde{\gamma}^*]).$$

- The closed-form expressions of \mathcal{R}_{ub}

$$\mathcal{R}_{ub} = \frac{1}{2} \log_2 (1 + 2\psi\sigma_D^2 \mathcal{Q}(-\mu_R/\sigma_R)).$$

The SNR/rate outage probability

- For the proposed system, the probability that the instantaneous SNR (γ) falls below a threshold SNR (γ_{th}) is referred as the SNR outage probability.
- An approximation to this SNR outage probability

$$P_{out} = \Pr(\gamma^* \leq \gamma_{th}) \approx F_{\gamma^*}(\gamma_{th}).$$

- The rate outage probability can also be readily obtained as

$$\begin{aligned} P_{out} &= \Pr(\mathcal{R}' \leq \mathcal{R}_{th}) = \Pr(\gamma^* \leq 2^{2\mathcal{R}_{th}} - 1) \\ &\approx F_{\gamma^*}(2^{2\mathcal{R}_{th}} - 1), \end{aligned}$$

where $\mathcal{R}' = \frac{1}{2} \log_2(1 + \gamma^*)$ is the achievable rate, and \mathcal{R}_{th} denotes a threshold rate.

The average symbol error rate (SER)

- The average SER of the proposed systems is defined as the expectation of conditional error probability ($P_{e|\gamma^*}$) over the probability distribution of γ^* .
- $P_{e|\gamma^*}$ is given for a broad range of coherent modulation schemes by $P_{e|\gamma^*} = \omega \mathcal{Q}(\sqrt{\vartheta\gamma^*})$, where the modulation scheme determines the values of fixed parameters ω and ϑ .
- Thus, the average SER: $\bar{P}_e = \mathbb{E}[\omega \mathcal{Q}(\sqrt{\vartheta\gamma^*})]$.
- A tight approximation for \bar{P}_e can be given as

$$\bar{P}_e \approx \frac{\omega}{2} - \frac{\omega\sqrt{\vartheta}}{2\sqrt{2\pi}} \int_0^\infty x^{-1/2} \exp\left(-\frac{\vartheta x}{2}\right) \bar{F}_{\tilde{\gamma}^*}(x) dx,$$

where $\bar{F}_{\tilde{\gamma}^*}(x) = 1 - F_{\tilde{\gamma}^*}(x)$ is the CCDF of γ^* .

- The closed-form solution to \bar{P}_e

$$\begin{aligned} \bar{P}_e \approx & \frac{\omega}{2} + 2\lambda \mathcal{Q}(-\mu_R/\sigma_R) + \frac{\lambda}{\pi\sqrt{a\sigma_R^2}} e^{\frac{-\mu_R^2}{2\sigma_R^2}} \left(1 - \frac{1}{2a\rho\sigma_R^2}\right) \\ & \times \sum_{i \in C_\infty} (-2)^i \left(\frac{\sqrt{a}\rho^2\sigma_R^2}{\mu_R}\right)^{i+1} \Gamma\left(\frac{i+1}{2}, \frac{\mu_R^2}{2a\rho\sigma_R^4}\right). \end{aligned}$$

Effects of Phase Quantization

- In practice, an IRS reflecting element only uses a set of discrete phase-shifts due to the associated hardware limitations.
- We investigate the impact of phase-shift quantization assuming that a limited number of discrete phase-shift is available for selection at the n th IRS element as $\hat{\theta}_n^* = \pi \hat{q} / 2^{b-1}$,
 - b is the number of quantization bits
 - $\hat{q} = \underset{q \in \{0, \pm 1, \dots, \pm 2^{b-1}\}}{\operatorname{argmin}} |\theta_n^* - \pi q / 2^{b-1}|$
 - $\hat{\theta}_n^*$ is the optimal phase-shift
- The difference between unquantized and quantized phase-shift is defined as the phase quantization error: $\epsilon_n = \theta_n^* - \hat{\theta}_n^*$.
- When the number of quantization levels increases, ϵ_n converges to a uniform distribution as $\epsilon_n \sim \mathcal{U}[-\pi/2^b, \pi/2^b)$.
- The optimal SNR with discrete phase-shift

$$\hat{\gamma}^* = \frac{\bar{\gamma}_R \bar{\gamma}_D \beta_{h^D}^2 \left(\sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I} e^{j\epsilon_n} \right)^2}{\bar{\gamma}_R \left(\sum_{n \in \mathcal{N}} \eta_n \beta_{h_n^R} \beta_{h_n^I} e^{j\epsilon_n} \right)^2 + \bar{\gamma}_D \beta_{h^D}^2 + 1}.$$

Simulation: Outage probability

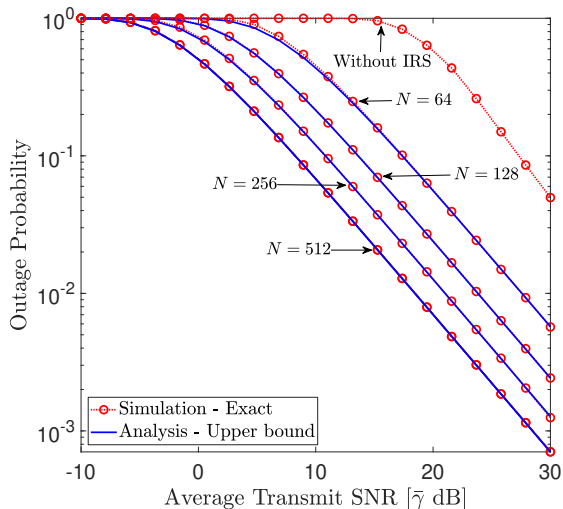


Figure: The outage probability for $N \in \{64, 128, 256, 512\}$ and the threshold SNR is $\gamma_{th} = 0$ dB. .

Simulation: Average achievable rate

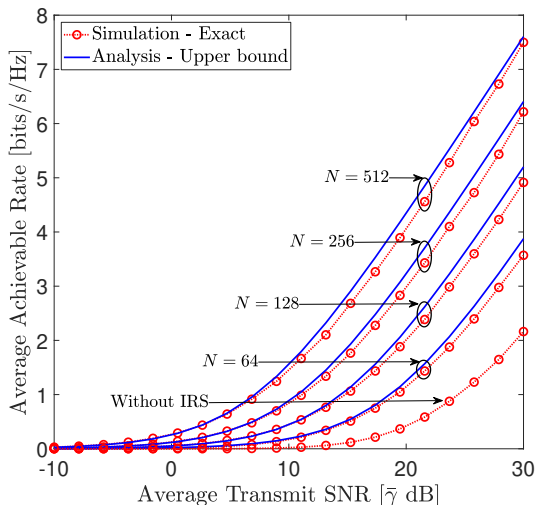


Figure: The average achievable rate for $N \in \{64, 128, 256, 512\}$.

Simulation: Average BER

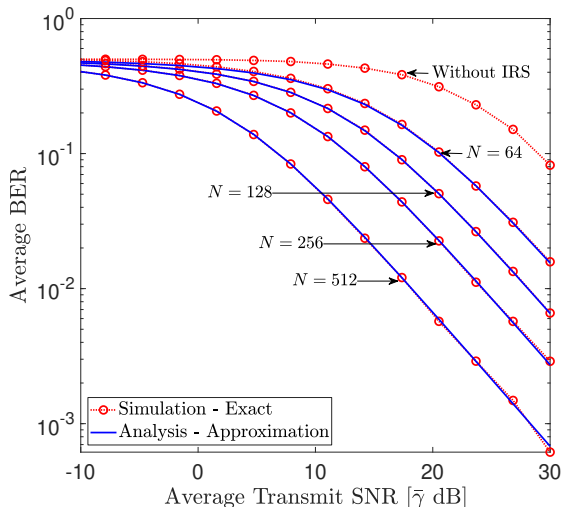


Figure: The average BER of BPSK for $N \in \{64, 128, 256, 512\}$, $\omega = 1$, and $\vartheta = 2$.

Simulation: Phase-shift quantization

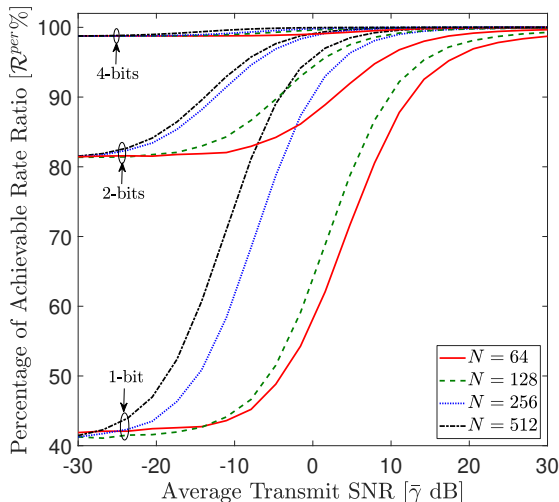


Figure: The effect of phase shift quantization on the average achievable rate for $N \in \{64, 128, 256, 512\}$.

Conclusions

- The performance of an IRS-assisted relay system has been investigated.
- The optimal SNR that is attained through intelligent phase-shift controlling of the IRS elements has been probabilistically characterized by deriving a tight CDF approximation.
- Thereby, tight approximations/bounds for the fundamental performance metrics, including the average achievable rate, SNR/rate outage probability, and average SER have been derived.
- The impact of phase-shift errors at the IRS has been investigated by adopting discrete phase-shift adjustments.
- The accuracy of our analysis has been validated through the Monte-Carlo simulation.
- A rigorous set of numerical results has been presented to investigate the performance of the proposed IRS-assisted relay system.
- From our numerical results, we reveal that the IRS-assisted relay systems can enhance the end-to-end wireless communication performance.

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Thank you for your attention!

Questions ??

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